Eur. Phys. J. C 7, 271–278 (1999)
DOI 10.1007/s100529801009

THE EUROPEAN PHYSICAL JOURNAL C

The η-η' mixing angle revisited

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Received: 14 May 1998 / Published online: 6 November 1998

Abstract. The value of the η-η' mixing angle θ_P is phenomenologically deduced from a rather exhaustive and up-to-date analysis of data including strong decays of tensor and higher-spin mesons, electromagnetic decays of vector and pseudoscalar mesons, J/ψ decays into a vector and a pseudoscalar meson, and other transitions. A value of θ_P between -17° and -13° is consistent with the present experimental evidence and the average $\theta_P = -15.5^\circ \pm 1.3^\circ$ seems to be favoured.

1 Introduction

The value of the η-η' mixing angle in the pseudoscalar-meson nonet has been discussed many times in the last thirty years. Quite possibly it has become one of the most interesting $SU(3)$-breaking hadronic parameters to measure since $SU(3)$ symmetry was proposed. In recent years three independent analyses have surveyed world data indirectly measuring this angle. A well known contribution to this discussion is the phenomenological analysis performed by Gilman and Kauffman [1] almost a decade ago. The approximate value $\theta_P \simeq -20^\circ$ (see Sect. II for notation and definitions) was proposed by these authors through a rather complete discussion of the experimental evidence available at that time. Another analysis by two of the present authors [2] concluded that a somewhat less negative value, $\theta_P = -14^\circ \pm 2^\circ$, seems to be favoured. A significant difference between these two independent analyses concerns the set of rich data on J/ψ decays into a vector and a pseudoscalar meson, $J/\psi \to VP$, which were included in the first analysis [1] but not in the second one [2]. Finally, the more recent discussion involving several channels performed by Ball, Frère and Tytgat [3] has led to θ_P between -20° and -17°.

Our purpose in the present paper is to obtain a new value of this η-η' mixing angle along the lines of the previous works. To this aim, we will perform a rather exhaustive and updated analysis using the available world data [4] and well established phenomenology on strong interaction decays of meson resonances into pseudoscalar pairs, electromagnetic decays of low mass mesons, J/ψ decays into a vector and a pseudoscalar meson, and other transitions. Our main assumptions are the validity of $SU(3)$ symmetry and, quite often, the stronger condition of nonet symmetry to relate the $SU(3)$-octet to the $SU(3)$-singlet. We also introduce $SU(3)$-breaking corrections in terms of constituent quark mass differences when their effects can be controlled and/or computed. In this sense, we define $\bar{m} \equiv (m_u + m_d)/2$ and take $m_s/\bar{m} \simeq 1.45$ from previous phenomenological analyses [5]. Finally, we also assume the η-η' to form a simple two-state system and neglect possible mixing with other pseudoscalar states, in particular with glueballs; we therefore consider just one single and real mixing angle.

Notice however that, according to recent analyses in the context of Chiral Perturbation Theory (ChPT), a description of the η-η' system beyond leading order cannot be achieved in terms of just one angle. A physical reason for that is the emergence of a q^2-dependence when (chiral) loop corrections are computed. An excellent example of such behaviour can be found in the analog π^0-η mixing analysis by Maltman [6], where loop effects are seen to modify the mixing angle by some 8% when moving from $q^2 = m_\pi^2$ to $q^2 = m_\eta^2$. A naive extrapolation of these results to our η-η' case is, however, not reliable due to the higher mass of the η' meson and its non-Goldstone nature for a finite number of colors N_C. Progress on this issue has been achieved only very recently combining the usual ChPT series expansion with a second expansion in powers of $1/N_C$ [7–9]. The q^2-dependence expected from loop corrections starts, however, only at orders higher than those that can be actually computed due to the proliferation of unknown parameters in this nonet-extended ChPT approach. This justifies that most of our present analysis is based on a simple, one-angle mixing scheme. The exception should be the radiative $P \to \gamma\gamma$ annihilations, where a nonet-extended ChPT requires again two different mixing angles, $\theta_8 \neq \theta_0$, for the F_P decay constants at orders accessible to present day analyses [10,11].

The paper is organized as follows. In Sect. 2 we introduce the notation and interrelate quark content and mixing angles. Sects. 3 and 4 cover the strong decays into two pseudoscalars of spin-two (tensor) mesons, $T \to PP$, and higher-spin mesons, $M_J \to PP$ with $J = 3, 4 \ldots$, respectively. Electromagnetic radiative decays involving vector and pseudoscalar mesons, $V \to P\gamma$ and $P \to V\gamma$, are discussed in Sect. 5. In Sect. 6 we consider the two-photon annihilation decays $\pi^0, \eta, \eta\prime \to \gamma\gamma$. Sect. 7 deals with J/ψ decays into a vector and a pseudoscalar meson, $J/\psi \to VP$. Finally, in Sect. 8 we briefly present results on other transitions and in Sect. 9 we summarize our conclusions.

2 Notation

Throughout this section we fix our notation which follows quite closely that introduced by Gilman and Kaufman [1] and previous work by Rosner [12]. The $SU(3)$-octet and -singlet states are

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle \ , \quad |\eta_0\rangle = \frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle \ , \quad (1)$$

and, in terms of this $SU(3)$ basis, the physical η and $\eta\prime$ states are defined to be

$$\begin{aligned} |\eta\rangle &= \cos\theta_P|\eta_8\rangle - \sin\theta_P|\eta_0\rangle \ , \\ |\eta\prime\rangle &= \sin\theta_P|\eta_8\rangle + \cos\theta_P|\eta_0\rangle \ . \end{aligned} \quad (2)$$

For some purposes it is more convenient to use the so-called nonstrange(NS)-strange(S) quark basis:

$$\begin{aligned} |\eta\rangle &= X_\eta \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_\eta|s\bar{s}\rangle \\ &\equiv \cos\varphi_P|\eta_{NS}\rangle - \sin\varphi_P|\eta_S\rangle \ , \\ |\eta\prime\rangle &= X_{\eta\prime} \frac{1}{\sqrt{2}}|u\bar{u}+d\bar{d}\rangle + Y_{\eta\prime}|s\bar{s}\rangle \\ &\equiv \sin\varphi_P|\eta_{NS}\rangle + \cos\varphi_P|\eta_S\rangle \ , \end{aligned} \quad (3)$$

where $|\eta_{NS}\rangle = |u\bar{u}+d\bar{d}\rangle/\sqrt{2}$ and $|\eta_S\rangle = |s\bar{s}\rangle$. Assuming the orthogonality of the physical η-$\eta\prime$ states and no mixing with other pseudoscalars, one has

$$X_\eta^2 + Y_\eta^2 = X_{\eta\prime}^2 + Y_{\eta\prime}^2 = 1 \ , \quad X_\eta X_{\eta\prime} + Y_\eta Y_{\eta\prime} = 0 \ . \quad (4)$$

In this case, just a single and real mixing angle governs the whole mixing phenomena if any energy dependence is (as usual) neglected. In terms of θ_P or φ_P the X's and Y's can be written

$$\begin{aligned} X_\eta = Y_{\eta\prime} &\equiv \cos\varphi_P = \tfrac{1}{\sqrt{3}}\cos\theta_P - \sqrt{\tfrac{2}{3}}\sin\theta_P \ , \\ Y_\eta = -X_{\eta\prime} &\equiv -\sin\varphi_P = -\sqrt{\tfrac{2}{3}}\cos\theta_P - \tfrac{1}{\sqrt{3}}\sin\theta_P \ , \end{aligned} \quad (5)$$

with $\theta_P = \varphi_P - \arctan\sqrt{2} \simeq \varphi_P - 54.7°$ and, conversely,

$$\tan\theta_P = -\frac{\sqrt{2}X_\eta + Y_\eta}{X_\eta - \sqrt{2}Y_\eta} = \frac{X_{\eta\prime} - \sqrt{2}Y_{\eta\prime}}{\sqrt{2}X_{\eta\prime} + Y_{\eta\prime}} \ . \quad (6)$$

In most of the next sections, we start with the presentation of the phenomenological and $SU(3)$-symmetric lagrangians responsible for the different transitions. Then $SU(3)$-breaking effects controlled by constituent quark mass differences are introduced when their origin is understood and their effects can be computed. A common feature of these lagrangians is the appearance of the $SU(3)$ matrix P containing the fields of the pseudoscalar meson nonet and their derivatives. The normalization of the $SU(3)$ matrix P is such that its diagonal elements are $\pi^0/\sqrt{2} + \eta_8/\sqrt{6} + \eta_0/\sqrt{3}$, $-\pi^0/\sqrt{2} + \eta_8/\sqrt{6} + \eta_0/\sqrt{3}$ and $-2\eta_8/\sqrt{6}+\eta_0/\sqrt{3}$. Similar $SU(3)$ matrices V^μ, $T^{\mu\nu}$ ... are introduced for the nonets of vector, tensor and higher-spin mesons, and mixing phenomena inside these nonets are consistently taken into account. Physical amplitudes are extracted and the corresponding theoretical decay widths are computed and compared with the available data. As a result of the corresponding fits, independent estimates of the η-$\eta\prime$ mixing angle are obtained and discussed in each section.

3 Strong decays of tensor mesons $T(2^{++}) \to PP$

The phenomenological and $SU(3)$-symmetric lagrangian for these $T \to PP$ decays is

$$\begin{aligned} \mathcal{L}_{TPP} &= g\, tr(T^{\mu\nu}\{P, \partial_\mu\partial_\nu P\}_+) \\ &= g\, tr(T^{\mu\nu}(P\partial_\mu\partial_\nu P + (\partial_\mu\partial_\nu P)P)) \ , \end{aligned} \quad (7)$$

where P and $T^{\mu\nu}$ are the $SU(3)$-nonet matrices mentioned in the previous section and g is a generic strong-interaction coupling constant. Similarly, we define the f-$f\prime$ mixing angle in this tensor-meson nonet ($J^{PC} = 2^{++}$) in a way analogous to the pseudoscalar case (see [4]):

$$\begin{aligned} |f\rangle &= \cos\varphi_T|f_{NS}\rangle - \sin\varphi_T|f_S\rangle \ , \\ |f\prime\rangle &= \sin\varphi_T|f_{NS}\rangle + \cos\varphi_T|f_S\rangle \ , \end{aligned} \quad (8)$$

with $\varphi_T \equiv \varphi_2 = \theta_T^{PDG} - \arctan 1/\sqrt{2} \simeq 28° - 35.3° = -7.3°$. This small value for the mixing angle follows from the quadratic Gell-Mann–Okubo (GMO) mass formula [4] thus implying an almost ideal mixing in the tensor-meson nonet.

In Table 1 we present for each strong tensor-meson decay both the normalized coupling of the process and the experimental branching ratio. It is straightforward to obtain the theoretical decay amplitude and partial width

$$\Gamma(T \to PP) = \frac{g_{TPP}^2}{60\pi}\frac{|\mathbf{p}_P|^5}{m_T^2} \ , \quad (9)$$

where g_{TPP} is defined in Table 1, $\mathbf{p}_P$ is the momentum of the outgoing pseudoscalar meson and m_T is the mass of the decaying tensor resonance. The symmetry factor in case two identical pseudoscalar mesons were produced is included in the couplings. The couplings are assumed to be $SU(3)$ symmetric since in these decays one is not able to control the $SU(3)$ breaking corrections via $\bar{m}/m_s$.

Comparing the theoretical decay widths with the experimental data taken from [4] (see Table 1), we extract four independent determinations of the mixing angle φ_P.

Table 1. Strong decays of spin-two, tensor mesons into pseudoscalar pairs, $T(2^{++}) \to PP$. The three columns display the various decay modes, the corresponding coupling constants and the experimental branching ratios (BR) from [4], respectively. Consistent values for the mixing angle φ_P ($\theta_P \simeq \varphi_P - 54.7°$) are obtained by fitting the BR's of each separate tensor meson. Values for φ_T are simultaneously predicted

decay mode	$g_{TPP}/2g$	BR(%) mixing angle(s)
$a_2 \to K\bar{K}$	1	4.9 ± 0.8
$a_2 \to \eta\pi$	$\sqrt{2}\cos\varphi_P$	14.5 ± 1.2
$a_2 \to \eta\prime\pi$	$\sqrt{2}\sin\varphi_P$	0.57 ± 0.11
		$\varphi_P = 43.2° \pm 2.8°$
$K_2^* \to K\pi$	$\sqrt{3}/\sqrt{2}$	49.7 ± 1.2
$K_2^* \to K\eta$	$\frac{1}{\sqrt{2}}\cos\varphi_P - \sin\varphi_P$	$0.14^{+0.28}_{-0.09}$
		$\varphi_P = 40.7° \pm 3.7°$
$f \to \pi\pi$	$\sqrt{3}\cos\varphi_T$	$84.7^{+2.6}_{-1.2}$
$f \to K\bar{K}$	$\cos\varphi_T - \sqrt{2}\sin\varphi_T$	4.6 ± 0.5
$f \to \eta\eta$	$\cos\varphi_T\cos^2\varphi_P$ $-\sqrt{2}\sin\varphi_T\sin^2\varphi_P$	0.45 ± 0.10
		$\varphi_P = 42.7° \pm 5.4°$ $\varphi_T = -7.8° \pm 2.6°$
$f\prime \to \pi\pi$	$\sqrt{3}\sin\varphi_T$	0.82 ± 0.15
$f\prime \to K\bar{K}$	$\sin\varphi_T + \sqrt{2}\cos\varphi_T$	88.8 ± 3.1
$f\prime \to \eta\eta$	$\sin\varphi_T\cos^2\varphi_P$ $+\sqrt{2}\cos\varphi_T\sin^2\varphi_P$	10.3 ± 3.1
		$\varphi_P = 41.0° \pm 3.5°$ $\varphi_T = -2.3° \pm 0.2°$

Each determination is based on a fit performed with the same initial tensor resonance T decaying into different PP channels. In every case, the quality of the fits is very good and the errors in φ_P —coming only from the experimental error in the branching ratio (BR) but not from that on the total width of the decaying resonance— are quite small. These four independent determinations of φ_P are fully consistent. However two warnings are worthwhile: first, although the two values obtained for φ_T in Table 1 ($\varphi_T = -7.8° \pm 2.6°$ and $\varphi_T = -2.3° \pm 0.2°$) reasonably agree with the approximate value $\varphi_T \simeq -7.3°$ coming from the Gell-Mann–Okubo mass formula, they are slightly diverging; second, when trying to fit the experimental value $g_{f\to K\bar{K}}/g_{a_2\to K\bar{K}} = 1.51 \pm 0.15$ with the theoretically predicted ratio in the good $SU(3)$ limit $g_{f\to K\bar{K}}/g_{a_2\to K\bar{K}} = \cos\varphi_T - \sqrt{2}\sin\varphi_T$, one gets $\varphi_T = -25°^{+8°}_{-11°}$ a too negative value.

A global fit involving all the measured $T \to PP$ decays and thus requiring the corresponding experimental total width of the decaying tensor mesons has also been performed. It leads to $\varphi_P = 44.2° \pm 1.4°$ (or $\theta_P = -10.5° \pm 1.4°$) and to $\varphi_T \equiv \varphi_2 = -2.9° \pm 0.3°$. The quality of this global fit is much poorer (the χ^2 per degree of freedom is $\chi^2/d.o.f = 6.2$) than the previous partial fits as a consequence of the two warnings just mentioned.

In spite of this, one can conclude that the available data on strong $T \to PP$ decays seem to favour the value for the pseudoscalar mixing angle $\varphi_P \simeq 42°$ (or $\theta_P \simeq -13°$). This result confirms the conclusions presented in [1,2]. The $T \to PP$ decays were not considered in [3].

4 Other strong decays $M_J \to PP$, $J > 2$

In this section we discuss the strong interaction decays into pseudoscalar pairs of meson resonances with spin J higher than two, $M_J \to PP$. Following the standard nomenclature, these resonances belong either to the "normal" spin-parity series with $P = (-)^J$ or to the "abnormal" one. In the first case, one has $J^{PC} = 4^{++}, 6^{++}\ldots$ and the situation is similar to the 2^{++} case already discussed; in the second one, with $J^{PC} = 3^{--}, 5^{--}\ldots$ the similarities are with the well-known case of vector mesons, 1^{--}. The phenomenological lagrangian needed for both series of higher-spin meson decays is

$$\mathcal{L}_{M_J PP} = -i^J g\, tr(T^{\mu_1\mu_2\ldots\mu_J}\{P, \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_J}P\}_\pm)\ , \quad (10)$$

where $\{P, \partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_J}P\}_\pm$ stands for the anticommutator in case of even spin J (positive C) or commutator in case of odd spin J (negative C), as required by charge conjugation invariance. The previous lagrangians are taken to be nonet-symmetric because $SU(3)$-breaking effects linked to quark mass differences cannot be controlled. In Table 2 we show the coupling constants g_{JPP} for the decay processes we are interested in. It is then straightforward to calculate the theoretical decay rate

$$\Gamma(M_J \to PP) = \frac{g_{JPP}^2}{4\pi}\frac{J!}{2(2J+1)!!}\frac{|\mathbf{p}_P|^{2J+1}}{m_J^2}\ . \quad (11)$$

Concerning the experimental input, data on these high-spin mesons are rather scarce. In two cases, however, they can be useful to extract new values for the mixing angle φ_P. Indeed, the three measured branching ratios for $f_4(2044)$ lead to the values $\varphi_P = 41.2° \pm 3.7°$ (or $\theta_P = -13.5° \pm 3.7°$) and $\varphi_4 = 15.7° \pm 4.4°$ shown in Table 2 (φ_4 is defined as the mixing angle of the system $f_4(2044)$-$f_4(2220)$). Independently, the value $\varphi_P = 50° \pm 26°$ can be obtained from the two measured branching ratios of $K_3^*(1780)$. Notice that in this case one has a much larger error even if one started with the rather accurately measured branching ratio $BR(K_3^* \to K\eta/K_3^* \to K\pi)_{\exp} =$

Table 2. Strong decays of spin-three and spin-four mesons into pseudoscalar pairs, $M_J \to PP$. As in Table I, a value of the mixing angle φ_P is obtained from each set of BR's. The fit gives also a value for φ_4, the mixing angle in the spin-four nonet

decay mode	$g_{M_J PP}/2g$	BR(%) mixing angle(s)
$f_4 \to \pi\pi$	$\sqrt{3}\cos\varphi_4$	17.0 ± 1.5
$f_4 \to K\bar{K}$	$\cos\varphi_4 - \sqrt{2}\sin\varphi_4$	$0.68^{+0.34}_{-0.18}$
$f_4 \to \eta\eta$	$\cos\varphi_4\cos^2\varphi_P - \sqrt{2}\sin\varphi_4\sin^2\varphi_P$	0.21 ± 0.08
		$\varphi_P = 41.2° \pm 3.7°$
		$\varphi_4 = 15.7° \pm 4.4°$
$K_3^* \to K\pi$	$\sqrt{3}/\sqrt{2}$	19.3 ± 1.0
$K_3^* \to K\eta$	$\frac{1}{\sqrt{2}}\cos\varphi_P + \sin\varphi_P$	8.0 ± 1.5
		$\varphi_P = 50° \pm 26°$

0.41 ± 0.08. This is due to the fact that the theoretical ratio $BR(K_J^* \to K\eta/K_J^* \to K\pi) = 1/3(\cos\varphi_P + (-)^{J+1}\sqrt{2}\sin\varphi_P)^2(\mathbf{p}_\eta/\mathbf{p}_\pi)^{2J+1}$ contains the sign $(-)^{J+1}$ due to charge conjugation invariance. For the actual values of φ_P, this sign makes the dependence of this ratio on φ_P rather smooth for J odd, as we have just seen. On the contrary, that dependence is much stronger for J even, but then the ratio has to be very small and no data are known except for the case of K_2^*, as discussed in the previous section.

As a conclusion for this section, we can say that a pseudoscalar-mixing angle of $\varphi_P \simeq 41°$ (or $\theta_P \simeq -14°$) is favoured again from our simple $SU(3)$ analysis of $M_J \to PP$, $J > 2$, decays and particularly from those of $f_4(2050)$. This is a new result since these $M_J \to PP$ decays were not considered in previous analyses.

5 Radiative decays $V \to P\gamma$, $P \to V\gamma$

We start this section with the phenomenological lagrangian that conventionally accounts for the amplitudes of the decay processes $V \to P\gamma$ and $P \to V\gamma$

$$\mathcal{L}_{VP\gamma} = g\,\epsilon_{\mu\nu\alpha\beta}\,\partial^\mu A^\nu\, tr(Q(\partial^\alpha V^\beta P + P\partial^\alpha V^\beta))\ , \quad (12)$$

where g is a generic, electromagnetic coupling constant, $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor, A_μ is the photon field, P is the pseudoscalar meson matrix, V_μ its vector counterpart and Q is the quark-charge matrix $Q = diag\{2/3, -1/3, -1/3\}$. From the previous lagrangian, it is easy to calculate the theoretical decay widths

$$\Gamma(V \to P\gamma) = \frac{1}{3}\frac{g_{VP\gamma}^2}{4\pi}|\mathbf{p}_\gamma|^3 = \frac{1}{3}\Gamma(P \to V\gamma)\ , \quad (13)$$

where $g_{VP\gamma}$ is the specific coupling constant for each process defined in Table 3 and $|\mathbf{p}_\gamma|$ is the momentum of the final photon. We have computed all these transition amplitudes in the framework of the quark model with $SU(3)$ and nonet symmetry broken by constituent quark mass differences according to a well known and time-honored prescription. It amounts to a modification in the original charge quark matrix Q via the introduction of the multiplicative $SU(3)$-breaking term $1 - s_e \equiv \bar{m}/m_s \simeq 1/1.45$ in the s-quark charge entry, as required in these magnetic-dipolar transitions if one takes into account the well known differences between the light- and strange-quark magnetic moments. Contrasting with the two preceding sections, in the present case we can easily control and compute the effects of these corrections. Moreover, in our analysis, the apparently negligible effects of non-ideal mixing in the vector-meson nonet will be taken into account. Indeed, we introduce the small, but certainly non-vanishing,

Table 3. Radiative decays of light mesons, $V \to P\gamma$ and $P \to V\gamma$. Columns are organized as in the preceding Tables, but here $SU(3)$-breaking corrections are introduced in terms of constituent quark mass differences $\bar{m}/m_s \simeq 1/1.45$. The small mixing angle φ_V signalling departure of ω and ϕ from ideal mixing is not neglected and left as a free parameter in the fit. The resulting values for φ_P and φ_V are displayed. The value of the full widths used in the fit are: $\Gamma_\rho = 150.7 \pm 1.2$ MeV, $\Gamma_\omega = 8.43 \pm 0.10$ MeV, $\Gamma_\phi = 4.43 \pm 0.05$ MeV and $\Gamma_{\eta'} = 0.201 \pm 0.016$ MeV

decay mode	$g_{VP\gamma}/g$	BR(%) mixing angle(s)
$\rho^0 \to \eta\gamma$	$\cos\varphi_P$	$(3.8 \pm 0.7)\ 10^{-2}$
$\rho^0 \to \pi^0\gamma$	$1/3$	$(7.9 \pm 2.0)\ 10^{-2}$
$\rho^\pm \to \pi^\pm\gamma$	$1/3$	$(4.5 \pm 0.5)\ 10^{-2}$
$\omega \to \eta\gamma$	$\frac{1}{3}(\cos\varphi_P\cos\varphi_V - 2\frac{\bar{m}}{m_s}\sin\varphi_P\sin\varphi_V)$	$(8.3 \pm 2.1)\ 10^{-2}$
$\omega \to \pi^0\gamma$	$\cos\varphi_V$	8.5 ± 0.5
$\phi \to \eta\gamma$	$\frac{1}{3}(\cos\varphi_P\sin\varphi_V + 2\frac{\bar{m}}{m_s}\sin\varphi_P\cos\varphi_V)$	1.26 ± 0.06
$\phi \to \eta'\gamma$	$\frac{1}{3}(\sin\varphi_P\sin\varphi_V - 2\frac{\bar{m}}{m_s}\cos\varphi_P\cos\varphi_V)$	$< 4.1\ 10^{-2}$ CL=90%
$\phi \to \pi^0\gamma$	$\sin\varphi_V$	$(1.31 \pm 0.13)\ 10^{-1}$
$\eta' \to \rho\gamma$	$\sin\varphi_P$	30.2 ± 1.3
$\eta' \to \omega\gamma$	$\frac{1}{3}(\sin\varphi_P\cos\varphi_V + 2\frac{\bar{m}}{m_s}\cos\varphi_P\sin\varphi_V)$	3.02 ± 0.30
		$\varphi_P = 36.5° \pm 1.4°$
		$\varphi_V = 3.4° \pm 0.2°$

departure of ω and ϕ from the ideally mixed states $\omega_{NS} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi_S \equiv s\bar{s}$ by writing the physical states in the nonstrange-strange basis as

$$\begin{aligned} |\omega\rangle &= \cos\varphi_V |\omega_{NS}\rangle - \sin\varphi_V |\phi_S\rangle \ , \\ |\phi\rangle &= \sin\varphi_V |\omega_{NS}\rangle + \cos\varphi_V |\phi_S\rangle \ , \end{aligned} \tag{14}$$

where φ_V is a small angle signalling departure from ideal mixing. The absolute value and relative sign of the ω-ϕ mixing angle are well known, $\sin\varphi_V \simeq \tan\varphi_V = +0.059 \pm 0.004$ or $\varphi_V \simeq +3.4°$, and come from the clearly understood ratio [4,13] $\Gamma(\phi \to \pi^0\gamma)/\Gamma(\omega \to \pi^0\gamma) = \tan^2\phi_V (p_\phi/p_\omega)^3 = (8.10 \pm 0.94) \times 10^{-3}$ and the ω-ϕ interference effects measured in $e^+e^- \to \pi^+\pi^-\pi^0$ annihilation data [14,15]. However, in our analysis we have not fixed this angle to the above value but has been left as a free parameter to fit.

Table 3 displays all the decay channels involved in our discussion together with their theoretical amplitudes extracted from the lagrangian (12), as well as the experimental values for the respective decay widths taken from [4]. We have performed a global fit to all these decay widths in order to find out the most suitable η-$\eta\prime$ mixing angle. In addition, a fitted value of the ω-ϕ mixing angle is also obtained. The fit is excellent ($\chi^2/d.o.f = 1.4$) and the data seems to prefer the values $\varphi_P = 36.5° \pm 1.4°$ (or $\theta_P = -18.2° \pm 1.4°$) and $\varphi_V = 3.4° \pm 0.2°$. This value of φ_P nicely agrees with the ones proposed by Gilman and Kauffman [1] and by Ball *et al.* [3], but it is somewhat smaller than the one favoured in [2]. Concerning the value of φ_V it perfectly agrees with the one coming from the well known Gell-Mann–Okubo mass formula ($\varphi_V \simeq 39° - 35.3° = +3.7°$, see [4]) and (including the sign) with the previously mentioned values coming from radiative ω and ϕ decays and ω-ϕ interference in $e^+e^- \to \pi^+\pi^-\pi^0$ [14,15]. This agreement represents an important test of the correctness of our treatment.

Another, more crucial test, originally proposed by Rosner [12] and expected to be measured at DAΦNE ϕ-factory in the near future, to elucidate the definite value for φ_P is the measurement of the ratio

$$R_\phi \equiv \frac{\Gamma(\phi \to \eta\prime\gamma)}{\Gamma(\phi \to \eta\gamma)} = \cot\varphi_P^2 (1 - \frac{m_s}{\bar{m}} \frac{\tan\varphi_V}{\sin 2\varphi_P})^2 \left(\frac{p_{\eta\prime}}{p_\eta}\right)^3 . \tag{15}$$

This ratio predicts 7.6×10^{-3} for $\varphi_P = 35°$ ($\theta_P \simeq -20°$) and 5.6×10^{-3} for $\varphi_P = 39.2°$ ($\theta_P = -15.5°$), well within the expected capabilities of DAΦNE. A recent experimental measurement [16] of the branching ratio $BR(\phi \to \eta\prime\gamma) = 1.2^{+0.7}_{-0.5} \cdot 10^{-4}$ yields $R_\phi = 9.5^{+5.2}_{-4.0} \cdot 10^{-3}$, with an error still too large to decide between the previous predicted values.

6 $P^0 \to \gamma\gamma$

We begin the discussion giving the well known phenomenological lagrangian

$$\mathcal{L}_{P^0\gamma\gamma} = g\, \epsilon_{\mu\nu\alpha\beta}\, \partial^\mu A^\nu \partial^\alpha A^\beta\, tr(Q^2 P) \ , \tag{16}$$

Table 4. Two-photon annihilation decays π^0, η, $\eta\prime \to \gamma\gamma$. As in the previous Table, $SU(3)$-breaking effects are introduced and a new value for φ_P is obtained

decay mode	$g_{P^0\gamma\gamma}/g$	Decay width mixing angle
$\pi^0 \to \gamma\gamma$	$\frac{1}{3\sqrt{2}}$	7.74 ± 0.55 eV
$\eta \to \gamma\gamma$	$\frac{5}{9\sqrt{2}}(\cos\varphi_P - \frac{\sqrt{2}}{5}\frac{\bar{m}}{m_s}\sin\varphi_P)$	0.46 ± 0.04 keV
$\eta\prime \to \gamma\gamma$	$\frac{5}{9\sqrt{2}}(\sin\varphi_P + \frac{\sqrt{2}}{5}\frac{\bar{m}}{m_s}\cos\varphi_P)$	4.26 ± 0.19 keV
		$\varphi_P = 41.3° \pm 1.3°$

which describes the annihilation of a neutral pseudoscalar meson P^0 into two photons. In a straightforward manner one can extract from the previous lagrangian the theoretical decay rate for the various $P^0 \to \gamma\gamma$ processes

$$\Gamma(P^0 \to \gamma\gamma) = g_{P\gamma\gamma}^2 \frac{1}{64\pi} m_P^3 \ , \tag{17}$$

where $g_{P\gamma\gamma}$ is the coupling constant for each process presented in Table 4 and m_P is the mass of the decaying pseudoscalar meson. As in the previous section, $SU(3)$-breaking effects driven by the constituent quark mass ratio $\bar{m}/m_s$ can be controlled since they appear through a modification in the quark charge matrix Q similar to the previous case. For $\bar{m}/m_s \simeq 1/1.45$, a comparison of the theoretical decay rates of the processes $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$ and $\eta\prime \to \gamma\gamma$ with their experimental values is presented in Table 4. The result of the global fit leads to $\varphi_P = 41.3° \pm 1.3°$ (or $\theta_P = -13.4° \pm 1.3°$).

The quality of the fit is now marginally good ($\chi^2/d.o.f. = 3.9$). The value for φ_P presented here agrees with that obtained in [2] when quark-mass corrections were taken into account. However, our present value slightly disagrees with the one in [1], the main reason being the discrepancy existing in the ratio $\Gamma(\eta \to \gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma)$ used in [1] and its updated value (see [4]) used in our present discussion which is nearly 20% smaller. The independent analysis by Pham [17] for these processes lead to $\theta_P = -18.4° \pm 2.0°$.

In principle, these $P \to \gamma\gamma$ decay modes could also be studied in the context of ChPT as did, for instance, in [18], where most of the difficulties originated by the non-Goldstone nature of the $\eta\prime$-meson were at that time partially ignored (for a more recent attempt along similar lines leading to $\theta_P = -22.0° \pm 3.3°$, see [19]). But, as stated in the Introduction, the use of two mixing angles seems unavoidable at non-leading orders and already feasible for these $P \to \gamma\gamma$ annihilations proceeding through F_P decay constants. The marginal quality of our fit seems to confirm the need of this two-angle mixing scheme. One then obtains $\theta_8 \simeq -20°$ and $\theta_0 \simeq -4°$ [7], or $\theta_8 \simeq -22.2°$ and $\theta_0 \simeq -9.1°$ [10]. At higher orders one also gets $\theta_0 - \theta_8 \simeq 14°$ [11]. ChPT could also predict θ_P by means of pseudoscalar masses, but the situation is unclear as mentioned

Table 5. J/ψ decays into a vector and a pseudoscalar meson, $J/\psi \to VP$. A value of φ_P deduced from a partial fit including isospin $I = 1$ final states is shown. A detailed description of the parameters involved in the amplitudes, and details about the fit can be found in [20]

decay mode	$g_{J/\psi VP}$	BR(10^{-3}) mixing angle
$J/\psi \to \rho\eta$	$3e\cos\varphi_P$	0.193 ± 0.023
$J/\psi \to \rho\eta'$	$3e\sin\varphi_P$	0.105 ± 0.018
$J/\psi \to \omega_{NS}\pi^0$	$3e$	0.42 ± 0.06
		$\varphi_P = 40.2° \pm 2.8°$
$J/\psi \to \rho\pi$	$g + e$	12.8 ± 1.0
$J/\psi \to K^+\bar{K}^{*-}$	$g(1-s) + e(2-x)$	5.0 ± 0.4
$J/\psi \to K^0\bar{K}^{*0}$	$g(1-s) - 2e(1+x)/2$	4.2 ± 0.4
$J/\psi \to \omega_{NS}\eta$	$(g+e)X_\eta + \sqrt{2}rg(\sqrt{2}X_\eta + Y_\eta)$	1.58 ± 0.16
$J/\psi \to \omega_{NS}\eta'$	$(g+e)X_{\eta'} + \sqrt{2}rg(\sqrt{2}X_{\eta'} + Y_{\eta'})$	0.167 ± 0.025
$J/\psi \to \phi_S\eta$	$[g(1-2s) - 2ex]Y_\eta + rg(1-s)(\sqrt{2}X_\eta + Y_\eta)$	0.65 ± 0.07
$J/\psi \to \phi_S\eta'$	$[g(1-2s) - 2ex]Y_{\eta'} + rg(1-s)(\sqrt{2}X_{\eta'} + Y_{\eta'})$	0.33 ± 0.04
$J/\psi \to \phi_S\pi^0$	0	< 0.0068 CL=90%

in [20] and described in much more detail by Leutwyler ([21]).

7 J/ψ Decays

Here we discuss the value for the η-η' mixing angle that one can extract from the analyses of J/ψ decays into a vector plus a pseudoscalar, $J/\psi \to VP$. Previous studies of this subject have appeared in the literature for the last ten years. A first exhaustive analysis performed by the Mark III Collaboration [22] on the decays of J/ψ into VP concluded that the η and η' were both consistent in being composed only of up, down and strange quarks and yield to a value of $\theta_P = -19.2° \pm 1.4°$ for the pseudoscalar mixing angle. Another equally exhaustive analysis performed by the DM2 Collaboration [23] on the same $J/\psi \to VP$ decays reaches similar conclusions: the η and η' mesons are consistent with a pure $q\bar{q}$ structure and a value for the mixing angle of $\theta_P = -19.1° \pm 1.4°$ is obtained. Using only the data for the J/ψ decays into VP, Morosita *et al.* [24] obtained a value of $\theta_P = -20.2°$, but a more extensive analysis by the same authors including also the J/ψ decays into $P\gamma$ leads to the value $\theta_P = -18.3°$. Finally, a value of $\theta_P \sim -19°$ and the conclusion that gluonium contaminations do not seem to be present or, at least, are not required in the η-η' system was similarly defended in [25]. In summary, all these analyses unanimously favour a value of $\theta_P \simeq -19°$. A very recent work [20] performed by the present authors dealing with the same relevant set of $J/\psi \to VP$ decay data leads, however, to a value of $\theta_P = -16.9° \pm 1.7°$. This last analysis follows quite closely the just mentioned analyses in [22–24] except that the apparently negligible effects of non-ideal mixing in the vector-meson nonet, which turn out to be important, are fully taken into account in [20]

In this section we do not intend to repeat the complete and exhaustive analysis on J/ψ decays into VP performed in [20] but simply quote the main features and results. The relevant theoretical amplitudes and their corresponding experimental branching ratios can be seen in Table 5. The origin of the various terms in the different amplitudes and the definitions for the parameters involved are explained in detail in [20] but are essentially the same in all the previously mentioned earlier analyses. As stated before, however, our amplitudes in Table 5 refer to the unmixed states ω_{NS} and ϕ_S rather than to the physical, mixed states. The required physical amplitudes have to be obtained by means of (14). The three decay modes in the upper part of the table represent isospin-violating transitions between an isoscalar initial state and an isovector final one; they are driven by a common isospin-violating, electromagnetic amplitude e times a factor accounting for the quarks involved in each transition. The second part of the table lists transitions proceeding both through the isospin-violating amplitude e and to the isospin-conserving strong amplitudes g and rg associated with connected and disconnected gluonic diagrams, respectively (see [22,23] for details). Also $SU(3)$-breaking is taken into account through the parameters x and $s_e = 1 - \bar{m}/m_s$ (see [20]).

The main results of this analysis, already presented in [20], are the following:

i) using a simple and widely accepted model, an excellent partial fit ($\chi^2/d.o.f. = 0.7$) of the decays involving a final state with isospin $I = 1$ ($J/\psi \to \rho\eta$, $\rho\eta\prime$, $\omega\pi^0$) leads to a value of $\varphi_P = 40.2° \pm 2.8°$ (or $\theta_P = -14.5° \pm 2.8°$).

ii) a global fit to all the decay modes using a more sophisticated but also widely accepted model [20] leads similarly to $\varphi_P = 37.8° \pm 1.7°$ (or $\theta_P = -16.9° \pm 1.7°$). One also obtains an excellent value for the ω-ϕ mixing angle $\varphi_V = +3.5° \pm 2.2°$.

A value of $\varphi_P \simeq 39°$ seems to be favoured again by the cleanest subset of experimental data involving $I = 1$ final states. The global set of data (now including also $I = 0$ final states) is affected by smaller error bars and seems to confirm the same result although a much more complicated description is needed. As a conclusion, we can say that the whole analysis performed here improves previous analyses thanks to the introduction of a non-negligible ω-ϕ mixing angle φ_V, whose correct value is consistently reproduced when performing the fits. The value we have obtained, $\theta_P \simeq -16°$, is clearly favoured over those coming from the earlier analyses, $\theta_P \simeq -19°$.

8 Other transitions

In this section we discuss other processes related to the η-$\eta\prime$ mixing angle which have been considered by several authors. A classical example is the ratio between the reactions $\pi^- p \to \eta n$ and $\pi^- p \to \eta\prime n$ [1]. At very high energies the difference in phase space for the two processes becomes negligible and nonet-symmetry predicts the ratio of cross sections

$$\frac{\sigma(\pi^- p \to \eta\prime n)}{\sigma(\pi^- p \to \eta n)} = \tan^2 \varphi_P \ . \tag{18}$$

There exist some discrepancy concerning the experimental value of this ratio. For completeness, we quote the two early results already considered in [1]. One result [26] leads to $\varphi_P = 36.7° \pm 1.4°$ (or $\theta_P = -18.0° \pm 1.4°$) while the other [27] leads to $\varphi_P = 39.7° \pm 1.0°$ (or $\theta_P = -15° \pm 1°$). More recently, a dedicated analysis by the Crystal Barrel Collaboration [28] favors a mixing angle of $\varphi_P = 37.4° \pm 1.8°$ (or $\theta_P = -17.3° \pm 1.8°$).

Independent information comes from the recent analysis of semileptonic D_s decays [29] favouring a mixing angle in the range $-18° \leq \theta_P \leq -10°$ with the best agreement observed for $\theta_P = -14°$. Similarly, from the measurement of the $\pi^+\pi^-$ invariant-mass distribution in $\eta\prime \to \pi^+\pi^-\gamma$ [30] one can deduce, depending on the model, either $\theta_P = -16.44° \pm 1.20°$ ($\varphi_P = 38.30° \pm 1.20°$) or $\theta_P = -23.24° \pm 1.23°$ ($\varphi_P = 31.50° \pm 1.23°$) while a recent analysis of the η and $\eta\prime$ radiative decays into $\gamma l^+ l^-$ and $\gamma\pi^+\pi^-$ leads to $\theta_P \sim -16.5$ [31]. Finally, from the study of the photon-meson transition form factors [32] a value of $\theta_P = -16.7° \pm 2.8°$ has been determined.

One can safely conclude this miscellaneous section saying that $\theta_P \simeq -16°$ is favoured by all these recent and independent results.

9 Conclusions

We have made a rather exhaustive analysis of the pseudoscalar η-$\eta\prime$ mixing angle using well established and accepted phenomenology and the experimental data available at present. We have surveyed various types of data and found that the strong decays of tensor-mesons $T(2^{++}) \to PP$ and higher-spin mesons $M_J \to PP$, for which unfortunately one cannot account for $SU(3)$-breaking corrections, favour the choice of $\theta_P \simeq -13°$; essentially the same value is also favoured by the two-photon annihilation decays $P \to \gamma\gamma$. Other data such as the radiative decays $V \to P\gamma$ and $P \to V\gamma$, J/ψ decays into a vector and a pseudoscalar, together with other types of transitions favour the choice of $\theta_P \simeq -17°$. We should emphasize that our conclusions are based on the assumptions of the simple η-$\eta\prime$ mixing scenario, the use of the $SU(3)$ and nonet symmetry and the manner in which $SU(3)$-breaking corrections are introduced. In particular, all our fits have been performed with just one η-$\eta\prime$ mixing angle thus ignoring recent claims on the necessity of a second mixing angle coming from nonet-extended versions of ChPT. Although this issue deserves further analyses, a naive extension of our phenomenological fits to a two-angle scheme suggests that somewhat different values for the angle, $\theta_P \simeq -13°$ or $\theta_P \simeq -18°$, are prefered for processes involving the η or $\eta\prime$ mesons respectively.

In summary, we have just shown that present data are consistent with a mixing angle in the range of $\theta_P \simeq -17°$ and $\theta_P \simeq -13°$. A weighted average value of $\theta_P = -15.5° \pm 1.3°$ seems to be favoured by the different types of decays involved in the analysis.

Acknowledgements. The authors acknowledge partial support from the EEC-TMR program, Contract N. CT98-0169 and from CIRIT, Contract N. 96SGR-21. R. E. acknowledges support from I. I. S. N. (Belgium).

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